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# SIMPLE ONE DIMENSIONAL MODELS OF THE PENETRATION OF RODS

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This note discusses a simple one dimensional model for describing the penetration of armor by rods based upon work by Alekseevshii and Gogolewski. We show that these models give incorrect results because they assume that the mass of the target material is constant, not a function of time as it actually is. We show that assuming this time dependence of the mass, yields results quite consistent with experimental data.

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## SIMPLE ONE DIMENSIONAL MODELS
## OF THE PENETRATION OF RODS

There are a large number of simple, one-dimensional models which attempt to provide estimates of the penetration of rods in target materials. Some of the models become quite sophisticated. For example, the ARAP work attempts to include the effects of target heating in the flow about the intruding projectile.

Ray Gogolewski, TTO, DARPA, discussed a particularly simple model which purported to agree essentially with the data and which provided useful scaling laws. This model is based on Alekseevshii's work[1] and introduces the concept of mass erosion of the penetrator where $u$ is the erosion speed in the target and is related to $w$, the speed of the projectile by,



$$u = \frac{w}{1 + f} \quad , \tag{1}$$

where the parameter $f$ is given by

$$f = \left(\frac{\rho_t}{\rho_p}\right)^{1/2} \quad . \tag{2}$$

In Equation (2), $\rho_t$ is the target density and $\rho_p$, that of the penetrator.

Gogolewski's model begins with the conservation of momentum written as,

$$m \frac{dw}{dt} = -\sigma_p A \quad , \tag{3}$$

Where $\sigma_p$ is the yield stress of the rod, $A$ the cross sectional area and $m$ the mass which is decreasing with time due to erosion. This cannot be the correct momentum equation because the mass <u>is</u> varying with time. In the spirit of these simple models the momentum equation should read,

$$\frac{d}{dt} \, mw = -\sigma_\rho A \quad . \tag{4}$$

Let us discuss this more correct model and return to the Gogolewski approach, Eq. (3), later on.

Equation (4) is immediately integrated to give,

$$mw = m_o \, w_o \, (1 - \frac{t}{\tau}) \quad , \tag{5}$$

where $m_o$ and $w_o$ are the initial mass and speed of the penetrator and the time constant $\tau$ is,

$$\tau = \frac{m_o \, w_o}{\sigma_p \, A} \quad . \tag{6}$$

The rate of mass erosion is,

$$\frac{dm}{dt} = A\,\rho_p\,(w - u)$$

$$= -A\,\rho_p\,\frac{f}{1+f}\,w \quad . \tag{7}$$

Eq. (7) can be integrated with the help of Eq. (5) to give,

$$m^2 = m_o^2\left[1 - k\chi\left(1 - \frac{\chi}{2}\right)\right] \quad , \tag{8}$$

where $\chi \equiv t/\tau$ and the constant $k$ is,

$$k = 2\,\rho_p\,\frac{f}{1+f}\,\frac{w_o^2}{\sigma_p} \quad . \tag{9}$$

The penetration $P$ is given by

$$P = \int_o^t u\,dt \quad . \tag{10}$$

Performing this integral we get,

$$P = L\sqrt{\frac{\rho_p}{\rho_t}}\left\{1 - \sqrt{1 - \frac{f}{1+f}\;\rho_p\,\frac{w_o^2}{\sigma_p}}\right\} \quad , \tag{11}$$

where L is the original length of the penetrator.

This expression for penetration leads to the result,

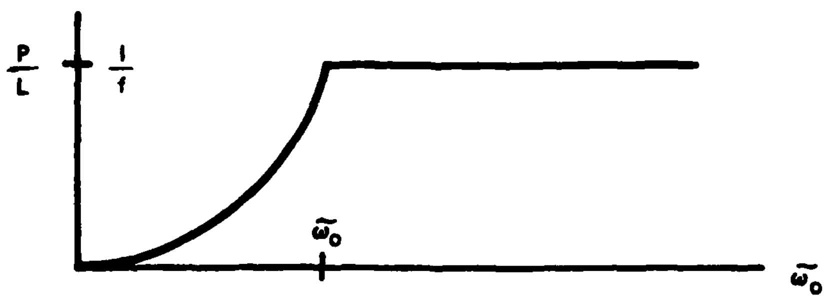
$$P \approx \frac{L}{2} \frac{\rho_p}{1+f} \frac{w_o^2}{\sigma_p} \quad , \quad w_o^2 << \frac{\sigma_p}{\rho_p} \quad , \tag{12}$$

or the penetration depth is proportional to the initial kinetic energy of the rod. This agrees with experiments in the speed domain indicated.

For very large initial speeds the mass erodes to zero before the speed of the rod goes to zero. Consequently, the penetration approaches the value,

$$P = L \sqrt{\frac{\rho_p}{\rho_t}} \quad , \quad w_o^2 >> \frac{\sigma_p}{\rho_p} \quad . \tag{13}$$

This also seems to be a well known limit which agrees with experiments in the indicated speed domain. Thus we expect the penetration as a function of initial speed to look like,



in agreement with the data presented.

The crossover speed $w_o^2$ is given by,

$$w_o = \left(\frac{1+f}{f} \; \frac{\sigma_p}{P_p}\right)^{1/2} \tag{14}$$

Eqs. (11), (12) and (13) give about as much information on scaling as one can reasonably expect from such a simple one-dimensional model.

Turning to the other momentum equation (Eq. (3)), one can show that the speed $w$ is related to the time $t$ by,

$$\exp\left(\frac{w_o^2}{2c_o^2}\right) \int_w^{w_o} \exp\left(\frac{w^2}{2c_o^2}\right) dw = \frac{\sigma_p A t}{m_o} \quad , \tag{15}$$

where

$$c_o^2 = \frac{1+f}{f} \; \frac{\sigma_p}{\rho_p} \quad . \tag{16}$$

Again, if $\dfrac{w_o^2}{2c_o^2} << 1$ we find,

$$P = \frac{1}{2} L \frac{\rho_p}{1+f} \; \frac{w_o^2}{\sigma_p} \quad , \tag{17}$$

as before. On the other hand, as $w_o$ increases, the penetration increases without bound which seems to contradict the data!

## LIST OF REFERENCES

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V. P. Alekseevshii, "Penetration of a Rod into a Target at High Velocity", Fizika Goreniya i Vzvyva, Vol. II, #2, p. 99-106, (1966), UDC532.501.32

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